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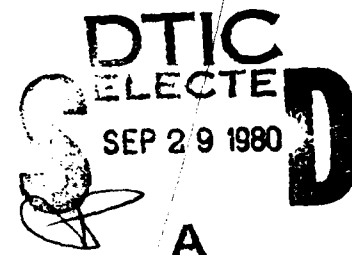
A COUNTEREXAMPLE FOR THE TROTTER
PRODUCT FORMULA

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ABSTRACT

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We exhibit here two linear m -accretive operators A_1 and A_2

whose sum is m -accretive but for which the associated product formulas

$$\left[S^{A_1} \left(\frac{t}{n} \right) S^{A_2} \left(\frac{t}{n} \right) \right]^n \quad \text{and} \quad \left[\left(I + \frac{t}{n} A_1 \right)^{-1} \left(I + \frac{t}{n} A_2 \right)^{-1} \right]^n \quad \text{do not converge.}$$

AMS (MOS) Subject Classifications: Primary 47H15; Secondary 34G05, 35K55

Key Words: m -accretive operators, semigroups of contraction, approximation

Work Unit Number 1 (Applied Analysis)

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SIGNIFICANCE AND EXPLANATION

A wide variety of partial differential equations as well as other equations can be written as ordinary differential equations of the form $u'(t) + Au(t) = 0$, where u takes values in a linear space X and A is an operator on X . The solution is given by $u(t) = S(t)u(0)$ where $S(t)$ is a semigroup of operators on X . In many cases the operator A can be written as the sum $A_1 + A_2$ of (possibly simpler) operators where A_1 and A_2 correspond to semigroups $S_1(t)$ and $S_2(t)$. Under appropriate conditions, the Trotter product formula $S(t)f = \lim_{n \rightarrow \infty} \left[S_1\left(\frac{t}{n}\right) S_2\left(\frac{t}{n}\right) \right]^n f$ relates $S(t)$ to $S_1(t)$ and $S_2(t)$ and provides one approach to the study of $S(t)$.

While various sufficient conditions for the validity of this limit are known, no satisfactory necessary conditions are known even when A_1 and A_2 are linear.

As part of the effort to understand the limitations on the validity of the product formula, we give an example in which A_1 , A_2 and $A_1 + A_2$ are all m -accretive but the corresponding semigroups do not satisfy the product formula.

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A COUNTEREXAMPLE FOR THE TROTTER PRODUCT FORMULA

Thomas G. Kurtz¹ and Michel Pierre^{1,2}

In [10], Trotter proved the following result: given $-A_1, -A_2$ the infinitesimal generators of two strongly continuous semigroups $S_1(t), S_2(t)$ of linear contractions on a Banach space X , if $-(\overline{A_1 + A_2})$ (the closure of $-(A_1 + A_2)$) is also the generator of such a semigroup, say $S_3(t)$, then, for any $f \in X$:

$$(1) \quad \forall t \in [0, \infty), \quad \lim_{n \rightarrow \infty} \left[S_1\left(\frac{t}{n}\right) S_2\left(\frac{t}{n}\right) \right]^n f = S_3(t)f.$$

Many attempts arose in the literature to extend this result to the case of nonlinear semigroups of contractions. In this context a natural question is: given A_1, A_2 two m -accretive operators on X such that $A_3 = A_1 + A_2$ is also m -accretive, is (1) true for the semigroups of contractions "generated" (in the sense of Crandall-Liggett [5]) by $-A_1, -A_2$ and $-A_3$ and for any $f \in \overline{D(A_3)}$ (assuming the product makes sense)?

A positive answer to this question has been provided with extra assumptions on A_1, A_2 or (and) on the space X , for instance the following:

- * A_1 and A_2 are continuous on X .
- * $-A_1$ is the generator of a linear contraction semigroup and A_2 is continuous on X .
- * X is a Hilbert space and $A_1, A_2, A_1 + A_2$ are single-valued maximal monotone operators (see Brézis-Pazy [2] or Brézis [1]).
- * X is a Hilbert space and A_1, A_2 are the subdifferentials of lower semi-continuous convex functions from X into $]-\infty, \infty]$ (see Masuda-Kato [7]).

Other results are also mentioned in Kato [6]. It is interesting to notice that all the results above are (more or less easy) consequences of the nonlinear version

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of Chernoff's lemma (see [3]) given by Brézis-Pazy in [2] which says: given

$(U(t))_{t \geq 0}$, a family of contractions from a closed convex subset C of X into itself, if there exists A_3 m -accretive such that $\overline{D(A_3)} = C$ and

$$\forall f \in C, \forall \lambda > 0, \lim_{t \rightarrow 0^+} \left[I + \frac{\lambda}{t} (I - U(t)) \right]^{-1} f = (I + \lambda A_3)^{-1} f,$$

then

$$\forall f \in C, \forall t \in [0, \infty], \lim_{n \rightarrow \infty} \left[U\left(\frac{t}{n}\right) \right]^n f = S_3(t)f.$$

The purpose of this paper is to give a counterexample showing that the question above has a negative answer in that general setting. Moreover we exhibit here two linear m -accretive operators A_1, A_2 whose sum $A_3 = A_1 + A_2$ is also m -accretive and for which (1) fails for some $f \in \overline{D(A_3)}$ as well as

$$\forall t \in [0, \infty), \lim_{n \rightarrow \infty} \left[\left(I + \frac{t}{n} A_1 \right)^{-1} \left(I + \frac{t}{n} A_2 \right)^{-1} \right]^n f = S_3(t)f.$$

To understand this counterexample with respect to Trotter's result, it is necessary to remember that an operator A on a Banach space X is said to be m -accretive if, for any $\lambda > 0$, $(I + \lambda A)^{-1}$ is a nonexpansive mapping defined on the whole space X (see e.g. [2] for more details). Consequently, by the well-known Hille-Yosida theorem, if A is a linear m -accretive operator, $-A$ is the (infinitesimal) generator of a strongly continuous semigroup of contractions if and only if its domain $D(A)$ is dense. Obviously this property fails in our examples below. Therefore, if these operators generate semigroups in the "nonlinear sense" (see Crandall-Liggett [5]), that is

$$(2) \quad \forall f \in \overline{D(A)}, \forall t \in [0, \infty), S(t)f = \lim_{n \rightarrow \infty} \left(I + \frac{t}{n} A \right)^{-n} f,$$

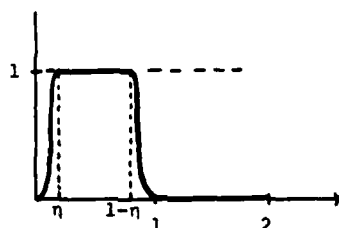
they are not strong generators of these semigroups.

Let $C_b(\mathbb{R})$ (resp. $C(K)$) denote the Banach space of the bounded continuous functions on \mathbb{R} (resp. on the compact set K of \mathbb{R}) with the norm

$$\forall u \in C_b(\mathbb{R}), \quad \|u\| = \sup_{x \in \mathbb{R}} |u(x)|$$

$$(\text{resp. } \forall u \in C(K), \quad \|u\| = \sup_{x \in K} |u(x)|).$$

Let $\rho \in C^\infty(\mathbb{R})$ be a periodic function with period 2 whose graph on $[0, 2]$ is:



On $C_b(\mathbb{R})$, we define the following operators (the derivative is taken in the sense of distributions).

$$(i) \quad D(A_1) = \{u \in C_b(\mathbb{R}); \quad \rho x^3 u' \in C_b(\mathbb{R})\}$$

$$A_1 u = \rho x^3 u'.$$

$$(ii) \quad D(A_2) = \{u \in C_b(\mathbb{R}); \quad (1 - \rho)x^3 u' \in C_b(\mathbb{R})\}$$

$$A_2 u = (1 - \rho)x^3 u'.$$

$$(iii) \quad D(A_3) = \{u \in C_b(\mathbb{R}); \quad x^3 u' \in C_b(\mathbb{R})\}$$

$$A_3 u = x^3 u'.$$

For any compact set K of \mathbb{R} , symmetric with respect to 0, we define on $C(K)$:

$$\forall i = 1, 2, 3, \quad D(A_i^K) = \{u \in C(K); \quad \alpha_i x^3 u' \in C(K)\}$$

$$A_i^K u = \alpha_i x^3 u',$$

where $\alpha_1 = \rho|_K$, $\alpha_2 = (1 - \rho)|_K$, $\alpha_3 = 1_K$. Here the derivative is taken in $D'(\overset{\circ}{K})$ and " $\alpha_i x^3 u' \in C(K)$ " means that $\alpha_i x^3 u'$ is continuous on $\overset{\circ}{K}$ and can be continuously extended to K .

PROPOSITION 1.

(i) For $i = 1, 2, 3$, $-A_i^K$ is the generator of a strongly continuous contraction semigroup S_i^K on $C(K)$ and $A_1^K + A_2^K = A_3^K$.

(ii) For $i = 1, 2, 3$, A_i is m -accretive on $C_b(\mathbb{R})$ and $A_1 + A_2 = A_3$.

(iii) For $i = 1, 2, 3$,

$$\forall f \in C_b(\mathbb{R}), \quad \forall \lambda > 0, \quad [(I + \lambda A_i)^{-1} f]_K = (I + \lambda A_i^K)^{-1} (f|_K).$$

(iv) If $S_i(t) : \overline{D(A_i)} \rightarrow \overline{D(A_i)}$ is defined by

$$\forall f \in \overline{D(A_i)}, \quad \forall t \geq 0, \quad S_i(t)f = \lim_{n \rightarrow \infty} \left[I + \frac{t}{n} A_i \right]^{-n} f,$$

then:

$$\forall f \in \overline{D(A_i)}, \quad \forall t \geq 0, \quad [S_i(t)f]_K = S_i^K(t)(f|_K).$$

Remark 1. If $u \in D(A_3)$, $x^3 u'$ is bounded. Hence $\lim_{x \rightarrow +\infty} u(x)$ and $\lim_{x \rightarrow -\infty} u(x)$ exist. Therefore $D(A_3)$ is not dense in $C_b(\mathbb{R})$.

Note also that, if $x_n, y_n \in [2n + \eta, 2n + 1 - \eta]$ and if $u \in D(A_1)$, then:

$$|u(x_n) - u(y_n)| \leq \frac{1}{2} \| \rho x^3 u' \| \left[\frac{1}{x_n^2} + \frac{1}{y_n^2} \right].$$

This also proves that $D(A_1)$ is not dense in $C_b(\mathbb{R})$.

PROPOSITION 2.

(i) $S_1(t)$ and $S_2(t)$ leave $\overline{D(A_3)}$ invariant and for all $f \in \overline{D(A_3)}$ and all $t \in [0, \infty)$, $\left[S_1\left(\frac{t}{n}\right) S_2\left(\frac{t}{n}\right) \right]^n f$ converges to $S_3(t)f$ uniformly on compact subsets of \mathbb{R} .

(ii) For all $f \in C_b(\mathbb{R})$ and all $t > 0$, $\left[\left(I + \frac{t}{n} A_1 \right)^{-1} \left(I + \frac{t}{n} A_2 \right)^{-1} \right]^n f$ converges to $S_3(t)f$ uniformly on compact subsets of \mathbb{R} .

But:

(iii) For any $f \in C_b(\mathbb{R})$ with compact support and $f \neq 0$, there exists $t \in (0, \infty)$ such that $\left[S_1\left(\frac{t}{n}\right) S_2\left(\frac{t}{n}\right) \right]^n f$ does not converge in $C_b(\mathbb{R})$.

For all $t \in]0, \infty)$, there exists $f \in C_b(\mathbb{R})$ such that $\left[S_1\left(\frac{t}{n}\right) S_2\left(\frac{t}{n}\right) \right]^n f$ does not converge in $C_b(\mathbb{R})$.

(iv) For any $f \in C_b(\mathbb{R})$ with compact support and $f \neq 0$, there exists t such that $\left[\left(I + \frac{t}{n} A_1 \right)^{-1} \left(I + \frac{t}{n} A_2 \right)^{-1} \right]^n f$ does not converge in $C_b(\mathbb{R})$.

Proof of Proposition 1.

The equalities $A_1^K + A_2^K = A_3^K$, $A_1 + A_2 = A_3$ follow directly from the definition.

For each $i = 1, 2, 3$, the proposition is a consequence of the following lemma.

Lemma. Let α be a nonnegative function of $C^\infty(\mathbb{R}) \cap C_b(\mathbb{R})$. Let A (resp. A^K) be defined on $C_b(\mathbb{R})$ (resp. $C(K)$) by

$$D(A) = \{u \in C_b(\mathbb{R}); \alpha x^3 u' \in C_b(\mathbb{R})\}, \quad Au = \alpha x^3 u'$$

$$(\text{resp. } D(A^K) = \{u \in C(K); \alpha x^3 u' \in C(K)\}, \quad A^K u = \alpha x^3 u').$$

Then:

(i) $-A^K$ is the generator of the strongly continuous semigroup of contractions $S^K(t)$ on $C(K)$ defined by

$$(3) \quad \forall f \in C(K), \quad S^K(t)f(x) = f(X(t, x)),$$

where $X(\cdot, x)$ is the solution of

$$(4) \quad \frac{d}{dt} X(t, x) = -\alpha(X(t, x))X^3(t, x), \quad X(0, x) = x.$$

Moreover, for all $\lambda > 0$

$$(5) \quad \forall f \in C(K), \quad \forall x \in K, \quad (I + \lambda A^K)^{-1} f(x) = \frac{1}{\lambda} \int_0^\infty e^{-\frac{t}{\lambda}} f(X(t, x)) dt.$$

(ii) A is m -accretive on $C_b(\mathbb{R})$ and

$$\forall f \in C_b(\mathbb{R}), \quad \forall x \in \mathbb{R}, \quad (I + \lambda A)^{-1} f(x) = \frac{1}{\lambda} \int_0^\infty e^{-\frac{t}{\lambda}} f(X(t, x)) dt,$$

$$\forall f \in \overline{D(A)}, \quad \forall x \in \mathbb{R}, \quad S(t)f(x) = f(X(t, x)),$$

where $S(t)$ is defined by (2).

Proof of the Lemma.

The proof of (i) is similar to the proof of Theorem (1.1) in [8].

Since K is symmetric and since $[X \mapsto -\alpha(X)X^3]$ is Lipschitz continuous on K and has the same sign as $-X$, (4) has a unique solution which stays in K for $x \in K$

and satisfies

$$\forall t \geq 0 \quad |X(t, x)| \leq |x|$$

$(t, x) \in [0, \infty[\times K \rightarrow X(t, x)$ is continuous.

It follows that (3) defines a strongly continuous semigroup of contractions $S^K(t)$ on $C(K)$ whose generator L is given by

$$Lu(x) = \lim_{t \rightarrow 0^+} \frac{u(X(t, x)) - u(x)}{t},$$

when the limit exists uniformly in $x \in K$. Proceeding as in [8], we prove that L is the closure of its restriction L_0 to $C^1(K)$. Indeed let L denote the Lipschitz continuous functions on K . Then, if $u \in D(L) \cap L$

$$Lu(x) = -\alpha(x)x^3 u'(x),$$

and $[u, Lu]$ is the limit in $C(K) \times C(K)$ of some $[u_n, L_0 u_n]$ with $u_n \in C^1(K)$. This proves that $\overline{L_0}$ contains the restriction of L to $D(L) \cap L$. But one can show that L is the closure of this restriction by using the fact that $S(t)$ leaves $D(L) \cap L$ invariant.

Now let us show $\overline{L_0} = A^K$. If $[u_n, \alpha x^3 u_n'] \in \overline{L_0}$ converges to $[u, v]$ in $C(K) \times C(K)$, then $\alpha x^3 u_n'$ converges to $\alpha x^3 u'$ in the sense of distributions; hence $\alpha x^3 u' = v \in C(K)$ which proves $\overline{L_0} \subset A^K$.

For the converse, as $I - \overline{L_0}$ is onto on $C(K)$, it is sufficient to remark that $I + A^K$ is one-one, that is:

$$(6) \quad (u \in C(K), u + \alpha x^3 u' = 0 \text{ in } D'(K)) \implies (u = 0 \text{ on } K).$$

This achieves the proof of (i), the property (5) being well-known.

To prove that A is m -accretive, let us consider for $f \in C_b(\mathbb{R})$ and $\lambda > 0$:

$$(7) \quad u_\lambda(x) = \frac{1}{\lambda} \int_0^\infty e^{-\frac{t}{\lambda}} f(X(t, x)) dt.$$

For any K as above, we have

$$\forall x \in K, u_\lambda(x) = (I + \lambda A^K)^{-1} (f|_K)(x).$$

As K is arbitrary, this proves that u_λ and $\alpha x^3 u'_\lambda$ are continuous on \mathbb{R} and verify

$$u_\lambda + \lambda \alpha x^3 u'_\lambda = f \text{ in } D'(\mathbb{R}).$$

Since $\|u_\lambda\| \leq \|f\|$ by definition, u_λ and $\alpha x^3 u'_\lambda \in C_b(\mathbb{R})$. Hence $u_\lambda \in D(A)$ and $u_\lambda + \lambda A u_\lambda = f$.

This proves that A is an extension of an m -accretive operator. Since $I + A$ is one-one (see (6)), A is m -accretive.

The relations (5) and (7) give

$$\forall f \in C_b(\mathbb{R}), [(I + \lambda A)^{-1} f]_K = (I + \lambda A^K)^{-1} (f|_K).$$

Hence, by the definition (2):

$$\forall f \in \overline{D(A)}, S(t)f|_K = \lim_{n \rightarrow \infty} [I + \frac{t}{n} A^K]^{-n} (f|_K) = S^K(t)(f|_K).$$

(The last equality is well-known for the linear generators.) Finally

$$\forall f \in \overline{D(A)} \quad S(t)f(x) = f(X(t, x)).$$

Remark 2. If $\alpha \equiv 1$ (i.e. $A = A_3$), we obtain that

$$X(t, x) = \frac{\operatorname{sgn} x}{\sqrt{2t + \frac{1}{x^2}}}.$$

Then, $\tilde{S}(t)f(x) = f(X(t, x))$ defines a semigroup of contractions on $C_b(\mathbb{R})$, but one can directly verify that $t \mapsto \tilde{S}(t)f$ is continuous at 0 if and only if

$f \in C(\overline{\mathbb{R}}) = \{g \in C_b(\mathbb{R}); \lim_{x \rightarrow \infty} g(x) \text{ and } \lim_{x \rightarrow -\infty} g(x) \text{ exist}\}$. Since $\tilde{S}(t)$ leaves $C(\overline{\mathbb{R}})$ invariant and since $\overline{D(A_3)} \subset C(\overline{\mathbb{R}})$ by the remark 1, $S_3(t)$ is exactly the restriction of $\tilde{S}(t)$ to $C(\overline{\mathbb{R}})$ and $C(\overline{\mathbb{R}}) = \overline{D(A_3)}$.

Proof of Proposition 2.

Observe that, by the definition of ρ , for $i = 1, 2$:

$$(8) \quad \begin{cases} \forall x > 0, & x - 1 - \eta \leq X_i(t, x) \leq x \\ \forall x < 0, & x \leq X_i(t, x) \leq x + 1 + \eta. \end{cases}$$

$(X_i, i = 1, 2,$ is the solution of (4) with $a_1 = 1, a_2 = 1 - \epsilon$. Therefore X_i is in $C_b(\mathbb{R})$ (which is the set $\{g \in C_b(\mathbb{R}); \lim_{x \rightarrow +\infty} g(x) \text{ and } \lim_{x \rightarrow -\infty} g(x) \text{ exist by Remark 1}\}$).

invariant under $S_1(t)$ and $S_2(t)$. Hence $\left[S_1\left(\frac{t}{n}\right)S_2\left(\frac{t}{n}\right)\right]^n f$ is defined for all $f \in C_b(\mathbb{R})$. Then, using (i), (iii) and (iv) in proposition 1, parts (i) and (ii) are consequences of Trotter and Chernoff's results (see [10], [3]).

Now by (8), if $f \in C_b(\mathbb{R})$ has compact support in $[-R, R]$, $S_1(t)f$ and $S_2(t)f$ also have compact support in $[-R - 1 - \eta, R + 1 + \eta]$ for any $t > 0$ and so do $(I + tA_1)^{-1}f$ and $(I + tA_2)^{-1}f$ by (ii) in the lemma.

So let $f \in C_b(\mathbb{R})$ have compact support and assume that $\left[S_1\left(\frac{t}{n}\right)S_2\left(\frac{t}{n}\right)\right]^n f$ or $\left[(I + \frac{t}{n}A_2)^{-1}(I + \frac{t}{n}A_1)^{-1}\right]^n f$ converge uniformly on \mathbb{R} . The limit is necessarily $S_3(t)f$ which is given by:

$$\forall t > 0, \forall x \neq 0, S_3(t)f(x) = f\left(\frac{\operatorname{sgn} x}{\sqrt{2t + \frac{1}{x^2}}}\right).$$

Then we have

$$0 = S_3(t)f(+\infty) = f\left(\frac{1}{\sqrt{2t}}\right), \quad 0 = S_3(t)f(-\infty) = f\left(\frac{-1}{\sqrt{2t}}\right).$$

If $f \neq 0$, this is false for some $t \in (0, \infty)$.

For the last statement of (iii), given $t > 0$, let $f \in C_b(\mathbb{R})$ have compact support and $f \equiv 1$ on $\left[-\frac{1}{\sqrt{2t}}, \frac{1}{\sqrt{2t}}\right]$. Then

$$S_3(t)f \equiv 1.$$

Clearly $\left[S_1\left(\frac{t}{n}\right)S_2\left(\frac{t}{n}\right)\right]^n f$, which has compact support, cannot converge uniformly to 1.

Remark 3. If $\hat{C}(\mathbb{R})$ denotes the continuous functions on \mathbb{R} which vanish at $\pm\infty$, let $\hat{A}_i = A_i \cap \hat{C}(\mathbb{R}) \times \hat{C}(\mathbb{R})$. Then we can show that $-\hat{A}_1, -\hat{A}_2$ are the (strong) generators of continuous semigroups of contractions $\hat{S}_1(t), \hat{S}_2(t)$. The same remarks as above prove that $\left[\hat{S}_1\left(\frac{t}{n}\right)\hat{S}_2\left(\frac{t}{n}\right)\right]^n f$ do not always converge in $\hat{C}(\mathbb{R})$. (Obviously

$-\hat{A}_3$ does not generate any semigroup even in the nonlinear sense.) Trotter also noted in [10] that the convergence of this product may fail for the sum of two generators.

Let us finally recall the example given by Pitt [9] showing that, if $-A_1, -A_2$ are two generators, the above product may converge even if $D(A_1) \cap D(A_2) = \{0\}$. See also Chernoff [4] for more pathological cases.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) THIS REPORT DESCRIBES we exhibit here two linear m-accretive operators A_1 and A_2 whose sum is m-accretive but for which the associated product formulas, $\left[S^{A_1}\left(\frac{t}{n}\right) S^{A_2}\left(\frac{t}{n}\right) \right]^n$ and $\left[\left(I + \frac{t}{n} A_1\right)^{-1} \left(I + \frac{t}{n} A_2\right)^{-1} \right]^n$ do not converge.		